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A Mathematical Modeling of Terrorism Dynamics and Control Strategies in Nigeria

Adamu Agabus Misal*[a] Aliyu Usman Kinafa [b] and Akinrefon Adesupo [c]

[a] *Department of Mathematics, Bingham University, Km 26 Abuja-Keffi Rd, New Karu, Nasarawa State, P.M.B 005, Nigeria.*
[b] *Department Mathematical Sciences, Gombe State University, Tudun Wada, Gombe, Gombe State, P.M.B 127, Nigeria.*
[c] *Department of Statistics, Modibbo Adama University, Sangerei Girei, Yola, Adamawa State, P.M.B 2076, Nigeria.*
misal4you@gmail.com

Abstract: A new Terrorism model was proposed having seven compartments: susceptible S(t), moderate M(t), terrorist T(t), combatant C(t), repentant R(t), rehabilitated $R_H(t)$, and detention D(t). The existence and positivity of the Terrorism model solution were proved, and local stability of the terrorism-free equilibrium as well as the endemic equilibrium of the terrorism model was established. Furthermore, the deterministic basic reproduction number (R_0) was computed from the system of ordinary differential equations formulated from the model using the next-generation matrix approach; we further investigated the stability of the model around the Terrorism free-equilibrium. It is shown that the model is locally asymptotically stable since R_0 is less than unity. Simulations conducted with the adaptive tau package in R revealed that while military intervention remains significant, it must be augmented with strategies targeting the root causes of radicalization. The findings emphasize the importance of dialogue and rehabilitation efforts, underscoring the critical role of non-kinetic measures in achieving sustainable stability. The study concludes by recommending an integrated strategy combining military actions with dialogue-based initiatives.

Keywords: Probability, Intervention, Dialogue. Radicalization, Modeling, Terrorism, Rehabilitation, Simulation

1. Introduction

The horrific events of September 11th, 2001, marked a turning point in the global fight against terrorism (Sandler, 2014). The emergence of groups like Boko Haram, a radical Islamist organization, further exacerbated the situation. The 2011 attack on the United Nations headquarters in Abuja tragically highlighted the evolving nature of the threat (Nossiter, 2011). The devastating consequences of terrorism in Nigeria are undeniable; the abduction of schoolgirls in 2014 and subsequent kidnapping incidents further underscored the brutality of these groups (Abdalla et al., 2017; Bolaji, 2018), resulting in the loss of life, destruction of property, and a climate of fear and insecurity (Chinwokwu, 2018). This rise in terrorism has not only displaced millions within Nigeria but also tarnished the country's international image (Abdu, 2018). Reports like the one from the African Insurance Organization in 2012 highlight the alarming prevalence of kidnappings for ransom, a lucrative criminal activity fueled by terrorism (Catlin, 2015).

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Nigeria continues to be one of the world's most terrorized countries, holding this grim position since 2015 according to the 2020 Global Terrorism Index (GTI). Only Iraq and Afghanistan are ranked worse. The human cost of this violence is staggering (Laden, 2012). Today, Nigeria grapples with over 3.3 million internally displaced persons, a stark reminder of the immense human cost of terrorism (IDMC, 2015).

Agent-based models (ABMs) have gained prominence for their ability to simulate the interactions between individuals, terrorist networks, and government agencies, providing insights into the spread of extremist ideologies and the impact of interventions (Smith & Johnson, 2021). These models have been particularly useful in understanding the resilience of groups like Boko Haram, which continue to adapt to military and policy responses (Okoli & Onuoha, 2022). Additionally, machine learning algorithms have been integrated with traditional mathematical frameworks to analyze large datasets, such as social media activity, to predict potential terrorist activities and identify recruitment patterns (Khan et al., 2023). Such advancements have enabled policymakers to design targeted strategies that address both the symptoms and root causes of terrorism.

In Nigeria, the application of mathematical models has been critical in addressing the persistent threat posed by Boko Haram and other extremist groups. Recent studies have utilized spatial-temporal models to map the geographic spread of terrorist activities, identifying regions at high risk of attacks and enabling proactive measures (Adeyemi et al., 2021). These models often incorporate data on population displacement, poverty levels, and political instability, reflecting the complex interplay of factors driving terrorism in the region (Idowu & Okafor, 2023). Furthermore, game-theoretic approaches have been employed to analyze the strategic interactions between terrorist groups and security forces, offering recommendations for optimal resource allocation and policy design (Eze et al., 2022). Despite these advancements, challenges remain, including data scarcity and the unpredictable nature of human behavior. Nevertheless, the integration of mathematical models with real-time data and advanced computational techniques holds promise for enhancing the effectiveness of counter-terrorism.

This study examines the transmission dynamics of terrorism in Nigeria, with the goal of developing more effective control measures. It seeks to enhance understanding of this complex issue, raise public awareness about the dangers of terrorism, and provide policymakers with actionable insights to strengthen their efforts against it. By analyzing the multifaceted nature of this challenge, the research aims to contribute meaningfully to the ongoing discourse on combating terrorism in Nigeria and beyond.

2. Materials and Method

2.1 Model Diagram and Equations

A model was developed to examine the transitions of individuals across various stages, incorporating factors such as recruitment, radicalization, counter-terrorism efforts, and rehabilitation programs. The dynamics of terrorism were represented as a continuous birth-death stochastic process. The model is visually depicted in the accompanying diagram. The model consists of Susceptible individuals (S) whom are likely to become a member of Boko Haram or to have their ideology in mind over time; when they come in contact with the Terrorist class (T). The Moderate class (M) or individuals are those who partially adopted their ideology but are not easily detected and they have not yet become a terrorist group. Some of the Terrorist class moved to the extreme and became combatants in exchange for fire from the military. The intervention of military strategies caused some of the terrorist groups to become Repentant (R) and are surrendered and moved to Rehabilitation or Radicalized class (R_h) and later may form part of the Susceptible class. The Population of the combatant class is then Detent (D) as depicted in the model diagram below.

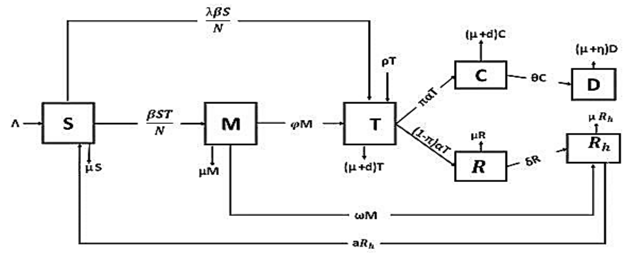

Figure 1: Model diagram with some interventions and control effect on the transmission dynamics of terrorism.

Where;

$$\frac{\delta S}{\delta t} = \Lambda + aR_H - \left(\frac{\beta\lambda}{N} + \frac{\beta T}{N} + \mu\right)S \quad (1)$$

$$\frac{\delta M}{\delta t} = \frac{\beta ST}{N} - (\omega + \varphi + \mu)M \quad (2)$$

$$\frac{\delta T}{\delta t} = \frac{\beta\lambda T}{N} + \varphi M + \rho T - (1 + d + \mu)T \quad (3)$$

$$\frac{\delta R}{\delta t} = \alpha(1-\pi)T - (\delta + \mu)R \quad (4)$$

$$\frac{\delta R_H}{\delta t} = \omega M + \delta R - (a + \mu)R_H \quad (5)$$

$$\frac{\delta C}{\delta t} = \alpha\pi T - (\theta + d + \mu)C \quad (6)$$

$$\frac{\delta D}{\delta t} = \theta C - (\eta + \mu)D \quad (7)$$

$$N = S + M + T + C + R + R_H + D$$

$$\frac{\delta N}{\delta t} = \frac{\delta S}{\delta t} + \frac{\delta M}{\delta t} + \frac{\delta T}{\delta t} + \frac{\delta C}{\delta t} + \frac{\delta R}{\delta t} + \frac{\delta R_H}{\delta t} + \frac{\delta D}{\delta t}$$

Hence; $$\frac{\delta N}{\delta t} = \Lambda - \mu N \quad (8)$$

2.2 Equilibrium Points and Stability Analysis

2.2.1 Positivity of the system

From the system of differential equations $(1)-(7)$, show boundedness and the positivity of the solution for the system of equations:

$$\left(S(0), M(0), T(0), C(0), R(0), R_H(0), D(0)\right) \in R^7. \quad (9)$$

For the system of equations $(1)-(7)$ show the region of attraction which is given by: Suppose $\Omega \subset R \times C^7$ is open $f_i \in \Omega \quad i = 1,2,3,\ldots 7$

Preposition (1);

The systems (1) – (7) are invariant of R^7 following the idea in (Cooke, 1996).

Proof

We have systems of equation $(1)-(7)$

$$\frac{\delta X}{\delta t}=F\left(X(t)\right),X(0)=0$$

$$\left\{X(t)\right\}=\left[F_1(x),F_2(x),\ldots,F_7(x),\right]^t$$

From the equation (1)

$$\frac{\delta S}{\delta t}=\Lambda+aR_H-\mu S-\frac{\beta\lambda S}{N}-\frac{\beta ST}{N},\quad \frac{\delta S}{\delta t}|_{S=0}=\Lambda+aR_H$$

Clearly,

$$\frac{\delta S}{\delta t}|_{S=0}=\Lambda+aR_H\geq 0$$

From equation (2)

$$\frac{\delta M}{\delta t}=\frac{\beta ST}{N}-\mu M-\omega M-\varphi M,\quad \frac{\delta M}{\delta t}|_{M=0}=\frac{\beta ST}{N}$$

Clearly, $\frac{\delta M}{\delta t}|_{M=0}=\frac{\beta ST}{N}\geq 0$

We continue the same way for (3), (4), (5), (6), and (7) from the preposition 1 above, the model of Terrorism is positive in the region. Hence, the system is invariant in the region. That satisfies the proof.

2.2.2 Boundedness of The System

Preposition 2; The system of equations (1) - (7) is bounded in the region.

Proof

From the system of differential equations (1) - (7)

$$\frac{\delta N}{\delta t}+\mu N=\Lambda \tag{10}$$

If the system is bounded in the invariant region, then,

$$\Omega=\left(S+M+T+C+R+R_H+D\right)\in R^7\,|\,(S+M+T+C+R+R_H+D<\Lambda)$$

From (10), it is clear that

$\frac{\delta N}{\delta t}\geq 0\forall t\geq 0$. Therefore, the system is bounded below by 0. That is, $\lim_{t\to 0}\inf\left(N(t)\right)\geq 0$

Again,

It is clear that

$$\frac{\delta N}{\delta t}\leq\Lambda$$

Therefore, it is bounded above by Λ which means

$\lim_{t\to\infty}SUP\left(N(t)\right)\geq\Lambda$. Hence, the system of equations is bounded in the region.

2.2.3 Existence and Positivity of the Solution

Here, the following results from the terrorist model given in equations (1) – (7) are well poised in the feasible region.

Preposition 3:

Let the initial condition be

$$\{S(0)>0,M(0)>0,T(0)>0,C(0)>0,R(0)>0,R_H(0)>0,D(0)>0,\}\in\Omega$$

is positive for all $t>0$. Then, the solution set $\left\{S,M,T,C,R,R_H,D\right\}(t)$ of the model system is positive for all $(t)>0$ using the idea in (Gao & Hethcote, 2006).)

Proof

From the equation (1)

$$\frac{\delta S}{\delta t}=\Lambda+aR_H-\mu S-\frac{\beta\lambda S}{N}-\frac{\beta ST}{N}$$

$$\frac{\delta S}{\delta t}\geq-\left[\mu S+\frac{\beta\lambda S}{N}+\frac{\beta ST}{N}\right]$$

$$\frac{\delta S}{\delta t}\geq-\left[\mu+\frac{\beta\lambda}{N}+\frac{\beta T}{N}\right]S$$

$$\frac{\delta S}{S}\geq-\left[\mu+\frac{\beta\lambda}{N}+\frac{\beta T}{N}\right]\delta t$$

$$\frac{\delta S}{S}\geq-\int\left[\mu+\frac{\beta\lambda}{N}+\frac{\beta T}{N}\right]\delta t$$

$$\ln S(t)\geq-\left[\mu+\frac{\beta\lambda}{N}+\frac{\beta T}{N}\right]t+C$$

Taking the exponent of both sides

$$\exp\ln S(t)\geq e^{-\left[\mu+\frac{\beta\lambda}{N}+\frac{\beta T}{N}\right]t+C}$$

$$S(t)\geq e^{\left[-(\mu+\frac{\beta\lambda}{N}+\frac{\beta T}{N}t+C)\right]}$$

Hence for equation (1), the solution exists

Observe that the exponential function is a positive function for all t

$(t)=0$, $S(t)|_{t=0}\geq 0$, $(t)\geq 0, \forall t\geq 0$, $S(t)\geq 0$ Hence, the solution for equation (1) exists and is positive.

From equation (2)

$$\frac{\delta M}{\delta t}=\frac{\beta ST}{N}-\mu M-\omega M-\varphi M$$

$$\frac{\delta M}{\delta t}\geq -(\mu+\omega+\varphi)M$$

$$\frac{\delta M}{M}\geq -(\mu+\omega+\varphi)\delta t$$

$$\int\frac{\delta M}{M}\geq -\int(\mu+\omega+\varphi)\delta t$$

$$\ln M(t)\geq -(\mu+\omega+\varphi)t+C$$

Taking the exponent

$$\exp\ln M(t)\geq e^{-(\mu+\omega+\varphi)t+C}$$

$$M(t)\geq Ce^{-(\mu+\omega+\varphi)t}$$

for equation (2), the solution exists

$$(t)=0,\ S(t)|_{t=0}\geq 0$$

$$(t)\geq 0, \forall t\geq 0$$

$$hence, S(t)\geq 0$$

Hence the solution for equation (2) exists and is positive. We continue in the same way and manner for equations (3), (4), (5), (6), and (7) we find that $T(t)\geq 0, C(t)\geq 0, R(t)\geq 0, R_H(t)\geq 0, D(t)\geq 0$ respectively, hence the solution set exists is positive for all $t\geq 0$.

2.2.4 Basic Reproduction Number (R_0)

The basic reproduction number R_0 is a measurement of the potential for spreading terrorism in a population. Mathematically, R_0 is a threshold parameter for the stability of a terrorist-free equilibrium and is related to the peak and the final size of an epidemic. It is defined as the expected number of secondary cases of infection that would occur due to a primary case in a completely susceptible population. If $R_0 < 1$, then the few infected individuals introduced into a completely susceptible population will, on average, fail to replace themselves, and terrorism will not spread. On the other hand, when $R_0 > 1$, then the number of infected individuals will increase with each generation and terrorism will spread.

The reproduction number can be computed using the next-generation operator method on the model equation. Using V and F notation for the new infection and the remaining transmission terms respectively.

$$V=\begin{bmatrix}\frac{\partial V_M}{\partial M}&\frac{\partial V_M}{\partial T}&\frac{\partial V_M}{\partial C}&\frac{\partial V_M}{\partial R}&\frac{\partial V_M}{\partial D}\\ \frac{\partial V_T}{\partial M}&\frac{\partial V_T}{\partial T}&\frac{\partial V_T}{\partial C}&\frac{\partial V_T}{\partial R}&\frac{\partial V_T}{\partial D}\\ \frac{\partial V_C}{\partial M}&\frac{\partial V_C}{\partial T}&\frac{\partial V_C}{\partial C}&\frac{\partial V_C}{\partial R}&\frac{\partial V_C}{\partial D}\\ \frac{\partial V_R}{\partial M}&\frac{\partial V_R}{\partial T}&\frac{\partial V_R}{\partial C}&\frac{\partial V_R}{\partial R}&\frac{\partial V_R}{\partial D}\\ \frac{\partial V_D}{\partial M}&\frac{\partial V_D}{\partial T}&\frac{\partial V_D}{\partial C}&\frac{\partial V_D}{\partial R}&\frac{\partial V_D}{\partial D}\end{bmatrix} \quad F=\begin{bmatrix}\frac{\partial F_M}{\partial M}&\frac{\partial F_M}{\partial T}&\frac{\partial F_M}{\partial C}&\frac{\partial F_M}{\partial R}&\frac{\partial F_M}{\partial D}\\ \frac{\partial F_T}{\partial M}&\frac{\partial F_T}{\partial T}&\frac{\partial F_T}{\partial C}&\frac{\partial F_T}{\partial R}&\frac{\partial F_T}{\partial D}\\ \frac{\partial F_C}{\partial M}&\frac{\partial F_C}{\partial T}&\frac{\partial F_C}{\partial C}&\frac{\partial F_C}{\partial R}&\frac{\partial F_C}{\partial D}\\ \frac{\partial F_R}{\partial M}&\frac{\partial F_R}{\partial T}&\frac{\partial F_R}{\partial C}&\frac{\partial F_R}{\partial R}&\frac{\partial F_R}{\partial D}\\ \frac{\partial F_D}{\partial M}&\frac{\partial F_D}{\partial T}&\frac{\partial F_D}{\partial C}&\frac{\partial F_D}{\partial R}&\frac{\partial F_D}{\partial D}\end{bmatrix}$$

$$V=\begin{bmatrix}A&0&0&0&0\\ -\varphi&B&0&0&0\\ 0&-\alpha\pi&C&0&0\\ 0&-\alpha(1-\pi)&0&D&0\\ 0&0&-\theta&0&E\end{bmatrix} \quad F=\begin{bmatrix}0&\frac{\beta}{N}&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\end{bmatrix}$$

$A=(\varphi+\omega+\mu), B=[\alpha\pi+(1-\pi)\alpha+\mu+d], C=(\theta+\mu+d), D=(\mu+d), E=(\eta+\mu)$

The next-generation matrix: Is the method used to drive R_0 for a compartmental model of the spread of terrorism. This method is given by (Diekmamn, 1990)

$$V^{-1}=\begin{bmatrix}\frac{1}{A}&0&0&0&0\\ \frac{\varphi}{AB}&\frac{1}{B}&0&0&0\\ \frac{\varphi\alpha\pi}{ABC}&\frac{\alpha\pi}{BC}&\frac{1}{C}&0&0\\ \frac{\alpha\varphi(1-\pi)}{ABD}&0&0&\frac{1}{D}&0\\ \frac{\varphi\alpha\theta\pi}{ABCE}&\frac{\alpha\theta\pi}{BCE}&\frac{0}{CE}&0&\frac{1}{E}\end{bmatrix}$$

$$FV^{-1}=\begin{bmatrix}\frac{\varphi\beta}{ABN}&\frac{\beta}{BN}&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\end{bmatrix}$$

The Basic reproduction number is denoted by $R_0=\Gamma(FV^{-1})$, Where Γ denotes the spectral radius, and gives

$$R_0=\frac{\varphi\beta}{ABN},\quad R_0=\frac{\beta}{N}\frac{1}{[\alpha\pi+(1-\pi)\alpha+\mu+d]}\cdot\frac{\varphi}{(\varphi+\omega+\mu)}$$

Where;

$\dfrac{\varphi}{(\varphi+\omega+\mu)}$ = the proportion of the exposed individuals that become terrorists.

$\dfrac{1}{[\alpha\pi+(1-\pi)\alpha+\mu+d]}$ = the average time an individual spends in the terrorist class before moving to the combatant or repentant class

$\dfrac{\beta}{N}$ = the Transmission proportion of the moderate and the Terrorist class.

2.2.5 Steady State of The Terrorist-Free Equilibrium

A steady state of Terrorist- free equilibrium is such that equations (1) – (7) are set to zero

$$\frac{\delta S}{\delta t}=0\Rightarrow \Lambda+aR_H-\left(\frac{\beta\lambda}{N}+\frac{\beta T}{N}+\mu\right)S=0$$

$$\frac{\delta M}{\delta t}=0\Rightarrow \frac{\beta ST}{N}-(\omega+\varphi+\mu)M=0$$

$$\frac{\delta T}{\delta t}=0\Rightarrow \frac{\beta\lambda T}{N}+\varphi M+\rho T-(1+d+\mu)T=0$$

$$\frac{\delta R}{\delta t}=0\Rightarrow \alpha(1-\pi)T-(\delta+\mu)R=0$$

$$\frac{\delta R_H}{\delta t}=0\Rightarrow \omega M+\delta R-(a+\mu)R_H=0$$

$$\frac{\delta C}{\delta t}=0\Rightarrow \alpha\pi T-(\theta+d+\mu)C=0$$

$$\frac{\delta D}{\delta t}=0\Rightarrow \theta C-(\eta+\mu)D=0$$

A steady state where there will be no terrorism is called the Terrorist- free equilibrium i.e. the point where

$M^*=0, T^*=0, C^*=0, R^*=0, R_H{}^*=0$.

Hence, the Terrorist- free equilibrium is

$$E^*=(S^*,M^*,T^*,C^*,R^*,R_H{}^*,D^*)=(\frac{\Lambda}{\mu},0,0,0,0,0,0)$$

Theorem

The terrorist-free equilibrium of the system is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$ (Brauer, 2012)

Proof

Consider the following Jacobian Matrix of the system of (1) – (7) at steady state. The Routh-Hurwitz criteria are used to establish asymptotic stability of equilibrium for non-linear system of differential equation. The Routh –Hurwitz criteria provide the necessary and sufficient condition for all roots of the characteristic polynomial to contain negative parts, therefore entailing asymptotic stability (Merkin, 1997). The local stability of the equilibrium may be determined by the Jacobian matrix gives

$$J(\frac{\Lambda}{\mu},0,0,0,0,0,0)=\begin{bmatrix}\frac{\partial S^*}{\partial S} & \frac{\partial S^*}{\partial M} & \frac{\partial S^*}{\partial T} & \frac{\partial S^*}{\partial C} & \frac{\partial S^*}{\partial R} & \frac{\partial S^*}{\partial R_H} & \frac{\partial S^*}{\partial D}\\ \frac{\partial M^*}{\partial S} & \frac{\partial M^*}{\partial M} & \frac{\partial M^*}{\partial T} & \frac{\partial M^*}{\partial C} & \frac{\partial M^*}{\partial R} & \frac{\partial M^*}{\partial R_H} & \frac{\partial M^*}{\partial D}\\ \frac{\partial T^*}{\partial S} & \frac{\partial T^*}{\partial M} & \frac{\partial T^*}{\partial T} & \frac{\partial T^*}{\partial C} & \frac{\partial T^*}{\partial R} & \frac{\partial T^*}{\partial R_H} & \frac{\partial T^*}{\partial D}\\ \frac{\partial C^*}{\partial S} & \frac{\partial C^*}{\partial M} & \frac{\partial C^*}{\partial T} & \frac{\partial C^*}{\partial C} & \frac{\partial C^*}{\partial R} & \frac{\partial C^*}{\partial R_H} & \frac{\partial C^*}{\partial D}\\ \frac{\partial R^*}{\partial S} & \frac{\partial R^*}{\partial M} & \frac{\partial R^*}{\partial T} & \frac{\partial R^*}{\partial C} & \frac{\partial R^*}{\partial R} & \frac{\partial R^*}{\partial R_H} & \frac{\partial R^*}{\partial D}\\ \frac{\partial R_H{}^*}{\partial S} & \frac{\partial R_H{}^*}{\partial M} & \frac{\partial R_H{}^*}{\partial T} & \frac{\partial R_H{}^*}{\partial C} & \frac{\partial R_H{}^*}{\partial R} & \frac{\partial R_H{}^*}{\partial R_H} & \frac{\partial R_H{}^*}{\partial D}\\ \frac{\partial D^*}{\partial S} & \frac{\partial D^*}{\partial M} & \frac{\partial D^*}{\partial T} & \frac{\partial D^*}{\partial C} & \frac{\partial D^*}{\partial R} & \frac{\partial D^*}{\partial R_H} & \frac{\partial D^*}{\partial D}\end{bmatrix}$$

$$J(\frac{\Lambda}{\mu},0,0,0,0,0,0)=\begin{bmatrix}-A & 0 & \frac{-\beta}{N} & 0 & 0 & a & 0\\ \lambda & -C & \frac{\beta}{N} & 0 & 0 & 0 & 0\\ 0 & \varphi & -D & 0 & 0 & 0 & 0\\ 0 & 0 & \alpha\pi & -E & 0 & 0 & 0\\ 0 & 0 & \alpha(1-\pi) & 0 & -F & 0 & 0\\ 0 & \omega & 0 & 0 & \delta & -G & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & -H\end{bmatrix}$$

Let

$A=(\beta T+\lambda+\mu), C=(\varphi+\omega+\mu), D=(1+\mu+d+\rho),$
$E=(\theta+\mu+d), F=(\mu+\delta), G=(a+\mu), H=(\mu+\eta)$

Reducing the above matrix through echelon form to obtain

$$J(\frac{\Lambda}{\mu},0,0,0,0,0,0)=\begin{bmatrix}-A & 0 & \frac{-\beta}{N} & 0 & 0 & a & 0\\ 0 & -C & K_1 & 0 & 0 & \frac{\alpha\beta T}{A} & 0\\ 0 & 0 & K_4 & 0 & 0 & K_3 & 0\\ 0 & 0 & 0 & -E & 0 & K_5 & 0\\ 0 & 0 & 0 & 0 & -F & K_6 & 0\\ 0 & 0 & 0 & 0 & 0 & K_7 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & -H\end{bmatrix}$$

Therefore, the Eigen-values are

$\lambda_1=-A,\ \lambda_2=-C,\ \lambda_3=K_4,\ \lambda_4=-E,$

$\lambda_5=-F,\ \lambda_6=K_7,$

Clearly, all the Eigen-values are negative hence the terrorist free equilibrium point is locally asymptotically stable If $R_0 < 1$ and unstable if $R_0 > 1$

2.2.6 Endemic Equilibrium

The endemic equilibrium will be defined by $\phi = \beta T$ where ϕ is the force of infection βT

From

$$\varphi M + \frac{\beta \lambda S}{N} + \rho T - BT = 0$$

$$(B - \rho)T = \varphi M + \beta \lambda S$$

$$A\mu N(\alpha + \mu + d)T = \varphi \Lambda \beta T + \Lambda \lambda A$$

$$A\mu N(\alpha + \mu + d)T - \varphi \Lambda \beta T = \Lambda \lambda A$$

$$\left[A\mu N(\alpha + \mu + d) - \varphi \Lambda \beta\right]T = \Lambda \lambda A$$

$$T = \frac{\lambda \Lambda A}{\left[A\mu N(\alpha + \mu + d) - \varphi \Lambda \beta\right]}$$

Therefore,

$$K = \beta T, \quad K = \frac{\beta \lambda \Lambda(\varphi + \omega + \mu)}{A\mu N(\alpha + \mu + d)}$$

Therefore, for a unique R_0, the endemic equilibrium point will be obtained.

Table 1: Parameter interpretation

Parameter	*Interpretation*
Λ	Input rate (birth rate)
β	Contact rate
ρ	External recruitment
α	Intervention strategy.
a	Rate of recovery
δ	Rate of progression from Repentant to Rehabilitation
θ	Rate of Progression from Combatant to Detention Facility
φ	Rate of Recruitment
λ	Rate of recruitment from Susceptible to Terrorist
π	Proportion of movement from terrorist to the combatant or repentant compartment
ω	Rate of progression from Exposed to Rehabilitation
μ	Natural Death
d	Death due to terrorism
η	Death due to torture

Table 2: Parameter Values

Parameter	**Description**	**Value**	**Sources**
λ	Input rate (birth rate)	0.086	(Abdu & Okoro, 2016)
β	Contact rate	0.046	(Abdu & Okoro, 2016)
ρ	External recruitment	0.0005	(Gambo& Mohammed,2020)
α	Intervention strategy.	0.35	(Bolaji, 2018)
a	Rate of recovery	0.0005	(Laden, 2012)
δ	Rate of progression from Repentant to Rehabilitation	0.8	(Gamb& Mohammed,2020)
θ	Rate of Progression from Combatant to Detention Facility	0.25	(Gambo & Mohammed, 2020)
φ	Rate of Recruitment	0.07	(Gambo& Mohammed,2020)
Λ	Rate of recruitment from susceptible to terrorist	0.05	(Abdu & Okoro, 2016)
π	Proportion from terrorist to the combatant or repentant compartment	0.2	(Udoh & Oladeji, 2019)
ω	Rate of progression from exposed to Rehabilitation	0.03	(Laden, 2012)
μ	Natural Death	0.0034	(Abiodun et al, 2018)
d	Death due to terrorism	0.083	(Gambo& Mohammed,2020)
η	Death due to torture	0.025	(Gambo& Mohammed,2020)
S	Susceptible Compartment	10000	(NPC, 2020)
M	Exposed Compartment	2300	(Ministry of Defense, 2020)
T	Terrorist Compartment	80	(Ministry of Defense, 2020)
C	Combatant Compartment	5	(Ministry of Defense, 2020)
R	Repentant Compartment	80	(Ministry of Defense, 2020)
R_H	Rehabilitation Compartment	60	(Ministry of Defense, 2020)
D	Detention Compartment	2	(Ministry of Defense, 2020)

3.2 Results

The S M T C R R_H D model categorizes each individual into a hypothetical population belonging to one of seven compartments. We simulated the effect of control measures by varying the frequency of interactions between individuals in different compartments and the probability of transmitting terrorism during these interactions.

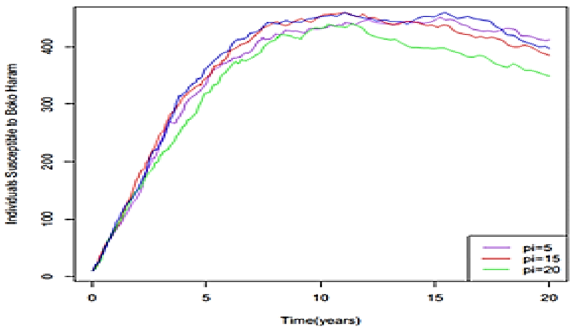

Figure 2: Population size of Susceptible class at varying level of recruitment rate

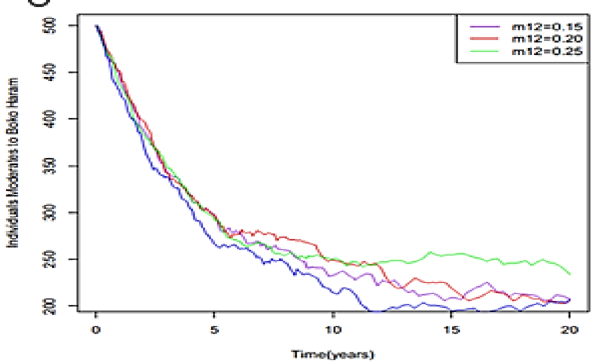

Figure 3:Population size of Moderate class at varying level of contact rate

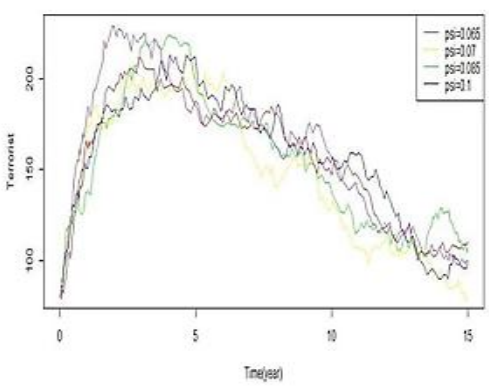

Figure 4:Population of Terrorism Class at varying level of ideological development

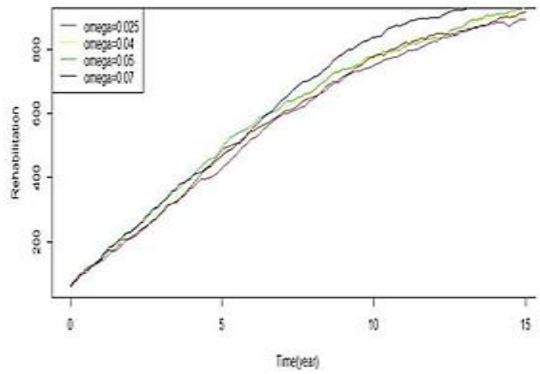

Figure 5: Population size of the Rehabilitated class at varying level of rehabilitation rate.

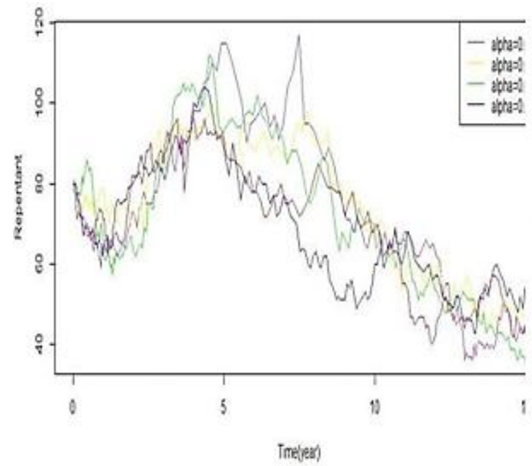

Figure 6: Population size of Repentant class at varying intervention rate.

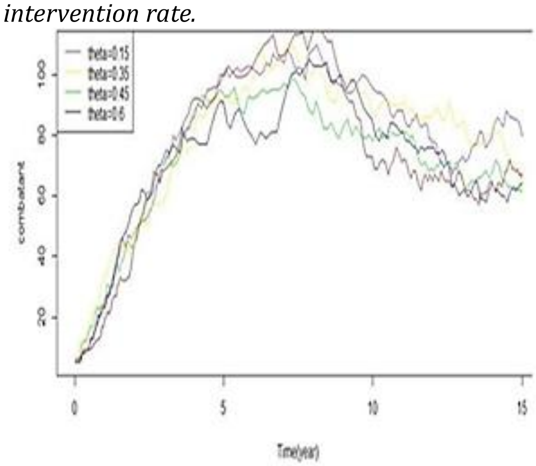

Figure 7: Population size of Combatant class at varying levels of Counter- Terrorist activity

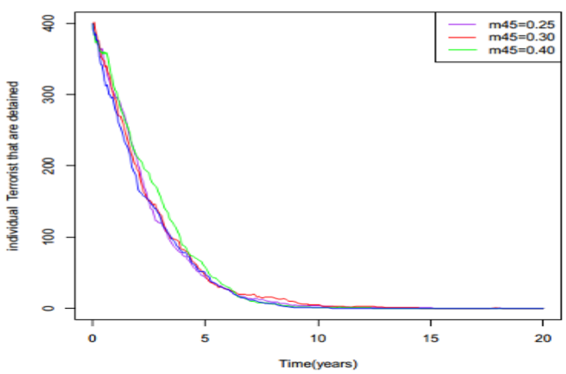

Figure 8: Population size of Detent class at varying level of counter terrorism.

3.2.1 Discussion

The findings presented in Figures 2 to 8 provide a detailed understanding of the dynamics of terrorism, the processes of radicalization, and the effectiveness of various intervention strategies. Figure 2 shows an initial rise in both the Susceptible population and the Terrorist compartment, peaking at year 4 when the contact rate reaches 0.45% terrorists per thousand susceptible individuals. This suggests that the early stages of terrorist activity are marked by rapid recruitment and ideological spread, particularly when susceptible individuals are highly exposed to extremist influences (Smith & Johnson, 2021). Figure 3 further emphasizes the role of terrorists in compelling susceptible individuals to adopt extremist ideologies to Moderate class, highlighting the importance of early intervention to disrupt recruitment networks (Adeyemi et al., 2021). Figure 4 reveals that the rate of full indoctrination into terrorist ideologies peaks at year 3, with 6.5% per thousand exposed individuals converting to terrorists. These peaks underscores the critical window for counter-radicalization efforts, as delays in intervention could lead to a significant increase in terrorist activities (Okoli & Onuoha, 2022). The subsequent decline in indoctrination rates after year 5, with a minor resurgence at year 13, suggests that sustained counter-terrorism measures are necessary to prevent long-term radicalization (Khan et al., 2023). Figures 5 to 8 highlight the impact of rehabilitation, intervention, and detention strategies in mitigating terrorism. Figure 5 demonstrates that the Rehabilitation compartment peaks at year 13 for a rehabilitation rate of 2.5% per thousand individuals, indicating that rehabilitation programs require time to yield significant results (Eze et al., 2022). Figure 6 shows that the Repentant compartment peaks at different intervals depending on the military intervention effort rates, emphasizing the need for adaptive strategies to address varying levels of terrorist activity (Idowu & Okafor, 2023). Figure 7 illustrates the effectiveness of intervention strategies on the Combatant compartment peaking at year 5 and year 8, suggesting that timely and consistent interventions can significantly reduce terrorist activities (Global Terrorism Index, 2023). Figure 8 indicates steady decline in the Detention class, reflecting the importance of law enforcement in curbing terrorism.

4. Conclusion

The research establishes the existence and positivity of solutions within the model and explores its stability under different conditions. The study concludes that the model effectively reflects real-world dynamics, particularly in assessing the impact of counter-terrorism measures in a specific case study of Boko Haram in Nigeria. It highlights that intervention strategies targeting both exposed individuals and active terrorists have proven effective in curbing the spread of terrorism in the north-eastern region of Nigeria. The research recommends future studies to address current limitations by broadening the model's scope and integrating additional variables and counter-terrorism strategies. This expansion will provide a deeper understanding of terrorism dynamics and facilitate the development of more robust solutions. Furthermore, the study underscores the critical role of public education and intelligence gathering in counter-terrorism efforts. Raising public awareness and collecting accurate intelligence are pivotal for addressing the root causes of terrorism and fostering the creation of more peaceful and resilient communities.

Authors' Contributions

A. A. Misal conceptualized and wrote the manuscript, A. U. Kinafa designed the study, and, conducted the analysis. A. Akinrefon conceptualized and supervised the work.

Declaration of Competing Interest

"The authors declare no competing interests."

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