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STOCHASTIC MODEL OF BOKO HARAM INSURGENCE WITH COUNTER TERRORIST OPERATIONS IN NORTH- EAST, NIGERIA

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ABSTRACT

The rise of Boko Haram activities in Northern Nigeria, particularly in the North-East, has led to frequent killings and the destruction of both public and private property, especially in areas like Yobe and Maiduguri. The safety of lives and property in this region is no longer guaranteed. In this study, we proposed and examined a stochastic model to understand and manage the dynamics of Boko Haram insurgency alongside counter-terrorism operations. We derived the differential equations that form the basis of the model and conducted simulations using the adaptivetau package in R. The findings indicate that counter-terrorism strategies have a significant effect on combating Boko Haram insurgencies. Consequently, we recommend that the Nigerian Army, with substantial support from the Air Force, intensify efforts to overcome the Boko Haram insurgency, including the acquisition of modern weaponry, the establishment of vigilante groups, and the implementation of rehabilitation programs to maintain the defeat of Boko Haram.

KEYWORDS: Stochastic, Boko- Haram, Counter- Terrorist

1 INTRODUCTION

Jama'atuAhlis Sunna Lidda'awatiwal-Jihad, commonly referred to as "Boko Haram," represents the most extreme form of violence in Nigeria's Fourth Republic. As of now, Nigeria is the third most affected country globally by terrorism, a ranking it has maintained since 2015. According to the 2020 Global Terrorism Index (GTI), only Iraq and Afghanistan have worse rankings than Nigeria, which has been grappling with violent insurgency in its northeastern region since 2009. While the majority of attacks occur in northern and North-East Nigeria, there have also been a considerable number of incidents in other areas. Notable attacks have taken place in Gombe, Kano, Kaduna, Jos, and Bauchi States, as well as in the Federal capital, Abuja, with the potential for more incidents in the future. Targets have included public venues where large groups of people gather, such as places of worship, markets, shopping centers, hotels, bars, restaurants, football viewing areas, displacement camps, transportation hubs, government buildings, and educational institutions (including schools, colleges, and universities), as well as international organizations. These attacks have often occurred during religious and public holidays in crowded locations, as well as during election periods [4]. The frequency and intensity of terrorism have risen, yet, despite the growing violence, terrorists continue to function within self-imposed limits. While terrorism is a global issue, its impact is not uniform across different regions. There is no singular reason that accounts for why certain societies experience higher rates of terrorist violence than others. Additionally, terrorists tend to use a restricted set of techniques that have seen little evolution over time [2].

The significant levels of terrorism and violence in Nigeria, perpetrated by fundamentalist groups such as ISWAP-Boko Haram, militants, and bandits, have intensified concerns among both the local population and the international community. This situation has severely impacted the economy, and the violence has transcended mere religious or political issues. Numerous meetings, summits, and conferences have been organized in an effort to address this crisis, but none have proven effective. Additionally, despite the federal government investing millions of naira to restore security in the country, these efforts have not yielded the desired results [4].

The group's ability to carry out attacks has become increasingly advanced, with evidence suggesting that some members may have been trained in bomb-making and other terrorist methods by Al-Qaeda-linked organizations in the northern and eastern regions of the continent. A series of assaults on churches between December 2011 and February 2012 indicates a deliberate strategy to provoke a widespread sectarian conflict aimed at destabilizing the country.

2 METHODOLOGY

We develop a stochastic model to analyze Boko Haram activities with counter-terrorism strategies. The behavior of Boko Haram was treated as a continuous-time birth-death stochastic process. This is represented by (X(t):t), where X(t) denotes the stochastic process, and the model is illustrated in the diagram below:

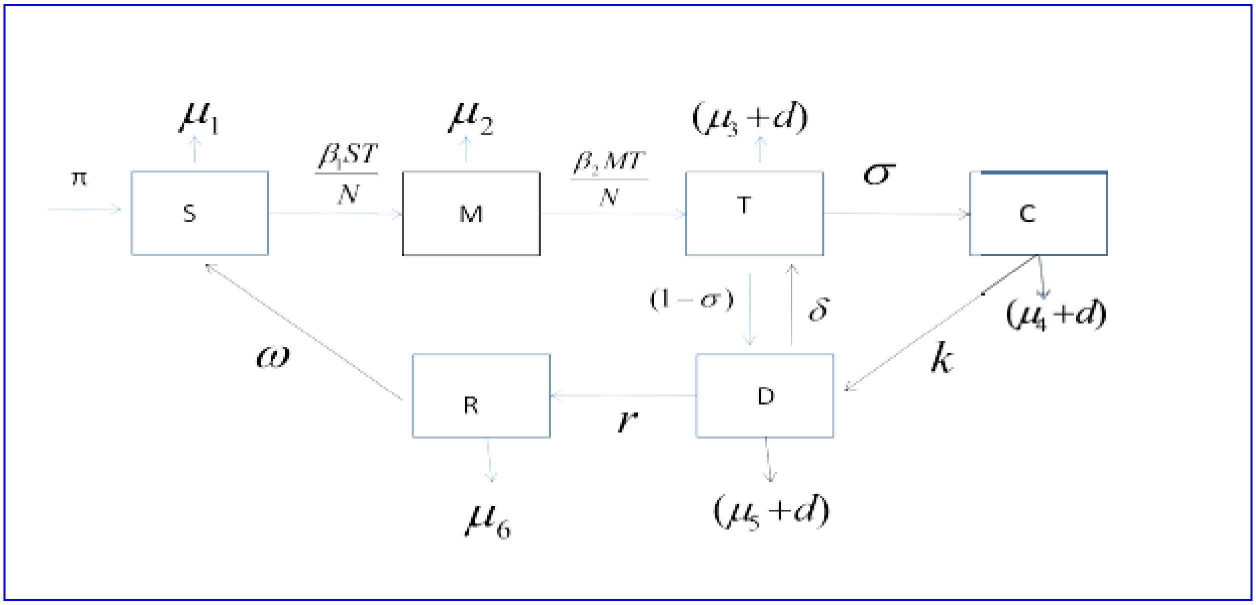

Figure 1: Model diagram

Based on the model presented above, we initially calculated the different transition probabilities by defining (X1 = S, X2 = M, X3 = T, X4 = C, X5 = D, X6 = R) as a collection of random variables that correspond to the states of Susceptible, Moderate, Terrorist, Combatant, Detention, and Repentance, respectively. The transition intensities and probabilities are summarized in the table below;

Table 1. Cumulative Transition Markov Chain

EVENT	VARIABLES	TRANSITIONS
$\pi\Delta(t)+o(\Delta t)$	$X_1+1, X_2, X_3, X_4, X_5, X_6$	1, 0, 0, 0, 0, 0
$m_{12}\Delta(t)+o(\Delta t)$	$X_1-1, X_2+1, X_3, X_4, X_5, X_6$	-1, +1, 0, 0, 0, 0
$m_{13}\Delta(t)+o(\Delta t)$	$X_1-1, X_2, X_3+1, X_4, X_5, X_6$	-1, 0, +1, 0, 0, 0
$m_{23}\Delta(t)+o(\Delta t)$	$X_1, X_2-1, X_3+1, X_4, X_5, X_6$	0, -1, +1, 0, 0, 0
$m_{34}\Delta(t)+o(\Delta t)$	$X_1, X_2, X_3-1, X_4+1, X_5, X_6$	0, 0, -1, +1, 0, 0
$m_{35}\Delta(t)+o(\Delta t)$	$X_1, X_2, X_3-1, X_4, X_5+1, X_6$	0, 0, -1, 0, +1, 0

$m_{36}\Delta(t) + o(\Delta t)$	$X_1, X_2, X_3-1, X_4, X_5, X_6+1$	0, 0, -1, 0, 0, +1
$m_{41}\Delta(t) + o(\Delta t)$	$X_1+1, X_2, X_3, X_4-1, X_5, X_6$	+1, 0, 0, -1, 0, 0
$m_{53}\Delta(t) + o(\Delta t)$	$X_1, X_2, X_3+1, X_4, X_5-1, X_6$	0, 0, +1, 0, -1, 0
$m_{56}\Delta(t) + o(\Delta t)$	$X_1, X_2, X_3, X_4, X_5-1, X_6+1$	0, 0, 0, 0, -1, +1
$m_{65}\Delta(t) + o(\Delta t)$	$X_1, X_2, X_3, X_4, X_5+1, X_6-1$	0, 0, 0, 0, +1, -1
$\mu_1\Delta(t) + o(\Delta t)$	$X_1 – 1$	-1, 0, 0, 0, 0, 0
$\mu_2\Delta(t) + o(\Delta t)$	$X_2 – 1$	0, -1, 0, 0, 0, 0
$\mu_3\Delta(t) + o(\Delta t)$	$X_3 – 1$	0, 0, -1, 0, 0, 0
$\mu_4\Delta(t) + o(\Delta t)$	$X_4 – 1$	0 ,0 ,0 -1, 0, 0
$\mu_5\Delta(t) + o(\Delta t)$	$X_5 – 1$	0, 0, 0, 0, -1, 0
$\mu_6\Delta(t) + o(\Delta t)$	$X_6 – 1$	0, 0, 0, 0, 0, -1

The probabilities of an event occurring as a result of transitions are determined using a series of difference equations referred to as Kolmogorov equations. In the context of continuous-time birth-death stochastic processes, we have:;

$$
\begin{aligned}
p_{x_1,\,x_2,\,x_3,\,x_4,\,x_5,\,x_6}(t+\Delta t) &= P\pi\Delta t \cdot p_{x_1+1}(t) + m_{12}(x_1-1)(x_2+1)\Delta t \cdot p_{x_1-1,x_2+1}(t) \\
&+ \mu_1(x_1+1)\Delta t \cdot p_{x_1-1,x_2+1}(t) + m_{13}(x_1-1)(x_3+1)\Delta t \cdot p_{x_1-1,x_3+1}(t) + \mu_1(x_1+1)\Delta t \cdot p_{x_1-1,x_3+1}(t) \\
&+ m_{23}(x_2-1)(x_3+1)\Delta t \cdot p_{x_2-1,x_3+1}(t) + \mu_2(x_2+1)\Delta t \cdot p_{x_2-1,x_3+1}(t) \\
&+ m_{34}(x_3-1)(x_4+1)\Delta t \cdot p_{x_3-1,x_4+1}(t) + \mu_3(x_3+1)\Delta t \cdot p_{x_3-1,x_4+1}(t) \\
&+ m_{35}(x_3-1)(x_5+1)\Delta t \cdot p_{x_3-1,x_5+1}(t) + \mu_3(x_3+1)\Delta t \cdot p_{x_3-1,x_5+1}(t) \\
&+ m_{36}(x_3-1)(x_6+1)\Delta t \cdot p_{x_3-1,x_6+1}(t) + \mu_3(x_3+1)\Delta t \cdot p_{x_3-1,x_6+1}(t) \\
&+ m_{41}(x_4-1)(x_1+1)\Delta t \cdot p_{x_4-1,x_1+1}(t) + \mu_4(x_4+1)\Delta t \cdot p_{x_4-1,x_1+1}(t) \\
&+ m_{53}(x_5-1)(x_3+1)\Delta t \cdot p_{x_5-1,x_3+1}(t) + \mu_5(x_5+1)\Delta t \cdot p_{x_5-1,x_3+1}(t) \\
&+ m_{56}(x_5-1)(x_6+1)\Delta t \cdot p_{x_5-1,x_6+1}(t) + \mu_5(x_5+1)\Delta t \cdot p_{x_5-1,x_6+1}(t) \\
&+ m_{63}(x_6-1)(x_3+1)\Delta t \cdot p_{x_6-1,x_3+1}(t) + \mu_6(x_6+1)\Delta t \cdot p_{x_6-1,x_3+1}(t) \\
&+ m_{65}(x_6-1)(x_5+1)\Delta t \cdot p_{x_6-1,x_5+1}(t) + \mu_6(x_6+1)\Delta t \cdot p_{x_6-1,x_5+1}(t) \\
&- \begin{bmatrix} 1 + P\pi + (\mu_1 + m_{12} + m_{13})(x_1+1) + (\mu_2 + m_{23})(x_2+1) \\ +(\mu_3 + m_{34} + m_{35} + m_{36})(x_3+1) + (\mu_4 + m_{41})(x_4+1) \\ +(\mu_5 + m_{53} + m_{56})(x_5+1) + (\mu_6 + m_{63} + m_{65})(x_6+1) \end{bmatrix} \cdot p_{x_1,\,x_2,\,x_3,\,x_4,\,x_5,\,x_6}(t)
\end{aligned}
\tag{1}
$$

Differentiating equation (1) using first principle, this gives us our Kolmogorov Forward Differential Equation below:

$$
\begin{aligned}
\frac{dp_{x_1,\,x_2,\,x_3,\,x_4,\,x_5,\,x_6}(t)}{dt} &= P\pi \square p_{x_1+1}(t) + m_{12}(x_1-1)(x_2+1)\square p_{x_1-1,x_2+1}(t) \\
&+\mu_1(x_1+1)\square p_{x_1-1,x_2+1}(t) + m_{13}(x_1-1)(x_3+1)\square p_{x_1-1,x_3+1}(t) + \mu_1(x_1+1)\square p_{x_1-1,x_3+1}(t) \\
&+m_{23}(x_2-1)(x_3+1)\square p_{x_2-1,x_3+1}(t) + \mu_2(x_2+1)\square p_{x_2-1,x_3+1}(t) \\
&+m_{34}(x_3-1)(x_4+1)\square p_{x_3-1,x_4+1}(t) + \mu_3(x_3+1)\square p_{x_3-1,x_4+1}(t) \\
&+m_{35}(x_3-1)(x_5+1)\square p_{x_3-1,x_5+1}(t) + \mu_3(x_3+1)\square p_{x_3-1,x_5+1}(t) \\
&+m_{36}(x_3-1)(x_6+1)\square p_{x_3-1,x_6+1}(t) + \mu_3(x_3+1)\square p_{x_3-1,x_6+1}(t) \\
&+m_{41}(x_4-1)(x_1+1)\square p_{x_4-1,x_1+1}(t) + \mu_4(x_4+1)\square p_{x_4-1,x_1+1}(t) \\
&+m_{53}(x_5-1)(x_3+1)\square p_{x_5-1,x_3+1}(t) + \mu_5(x_5+1)\square p_{x_5-1,x_3+1}(t) \\
&+m_{56}(x_5-1)(x_6+1)\square p_{x_5-1,x_6+1}(t) + \mu_5(x_5+1)\square p_{x_5-1,x_6+1}(t) \\
&+m_{63}(x_6-1)(x_3+1)\square p_{x_6-1,x_3+1}(t) + \mu_6(x_6+1)\square p_{x_6-1,x_3+1}(t) \\
&+m_{65}(x_6-1)(x_5+1)\square p_{x_6-1,x_5+1}(t) + \mu_6(x_6+1)\square p_{x_6-1,x_5+1}(t) \\
&-\begin{bmatrix} 1+P\pi+(\mu_1+m_{12}+m_{13})(x_1+1)+(\mu_2+m_{23})(x_2+1) \\ +(\mu_3+m_{34}+m_{35}+m_{36})(x_3+1)+(\mu_4+m_{41})(x_4+1) \\ +(\mu_5+m_{53}+m_{56})(x_5+1)+(\mu_6+m_{63}+m_{65})(x_6+1) \end{bmatrix}\square p_{x_1,\,x_2,\,x_3,\,x_4,\,x_5,\,x_6}(t)
\end{aligned}
\tag{2}
$$

Equation (2) is a full dynamics and it can be simplified or converted with the aid of multivariate probability generating function. We will then obtain the sets of the following differential equations for expectation below:

$$\frac{dE\left[X_1(t)\right]}{dt}=E\left[P\pi\right]+m_{41}E\left[X_4(t)X_1(t)\right]-m_{12}E\left[X_1(t)X_2(t)\right]-m_{13}E\left[X_1(t)X_3(t)\right]-\mu E\left[X_1(t)\right]$$

$$\frac{dE\left[X_2(t)\right]}{dt}=m_{12}E\left[X_1(t)X_2(t)\right]-m_{23}E\left[X_2(t)X_4(t)\right]-\mu E\left[X_2(t)\right]$$

$$\frac{dE\left[X_3(t)\right]}{dt}=m_{13}E\left[X_1(t)X_3(t)\right]+m_{23}E\left[X_2(t)X_4(t)\right]-m_{34}E\left[X_3(t)X_4(t)\right]-m_{35}E\left[X_3(t)X_5(t)\right]-m_{36}E\left[X_3(t)X_6(t)\right]-\mu E\left[X_3(t)\right]$$

$$\frac{dE\left[X_4(t)\right]}{dt}=m_{34}E\left[X_3(t)X_4(t)\right]-m_{41}E\left[X_4(t)X_1(t)\right]-\mu E\left[X_4(t)\right]$$

$$\frac{dE\left[X_5(t)\right]}{dt}=m_{35}E\left[X_3(t)X_5(t)\right]+m_{65}E\left[X_6(t)X_5(t)\right]-m_{53}E\left[X_5(t)X_3(t)\right]-m_{56}E\left[X_5(t)X_6(t)\right]-\mu E\left[X_5(t)\right]$$

$$\frac{dE\left[X_6(t)\right]}{dt}=m_{36}E\left[X_3(t)X_6(t)\right]+m_{56}E\left[X_5(t)X_6(t)\right]-m_{65}E\left[X_6(t)X_5(t)\right]-\mu E\left[X_6(t)\right]$$

(3)

Replacing X_1 = S, X_2 = M, X_3 = T, X_4 = A, X_5 = D and X_6 = C, the above model expectation equations can also be written as

$$\frac{dS(t)}{dt}=P\pi+m_{41}A(t)S(t)-m_{12}S(t)M(t)-\mu S(t)$$

$$\frac{dM(t)}{dt}=m_{12}S(t)M(t)-m_{23}M(t)T(t)-\mu M(t)$$

$$\frac{dT(t)}{dt}=m_{13}S(t)T(t)+m_{23}M(t)T(t)-m_{34}T(t)A(t)-m_{35}A(t)D(t)-m_{36}T(t)C(t)-\mu T(t)$$

$$\frac{dA(t)}{dt}=m_{34}T(t)A(t)-m_{41}A(t)S(t)-\mu A(t)$$

$$\frac{dD(t)}{dt}=m_{35}T(t)D(t)+m_{65}C(t)D(t)-m_{53}D(t)T(t)-m_{56}D(t)C(t)-\mu D(t)$$

$$\frac{dC(t)}{dt}=m_{36}T(t)C(t)+m_{65}D(t)C(t)-m_{56}C(t)D(t)-\mu C(t)$$

(4)

3 NUMERICAL SIMULATIONS

In this section, we analyze the stochastic of terrorism with Counter terrorist strategies. To solve our equation (4), we perform numerical simulation using initial values for state variables and transition parameters in Table 2 below.

Table 2: Parameters/ Variables Description, Source & Value

Variables/Parameters	Description	Value	Source
S	Susceptible Population	10,000	Estimated
M	Moderate Members	50	[5]
T	Terrorist	300	[5]
C	Combatant	80	[5]
D	Detention	20	[5]
R	Rehabilitation	35	[5]
π	Recruitment rate	10	[5]
β_1	Rate at which susceptible moved to Moderate class	0.01	[8]
β_2	Rate at which Moderate moved to Terrorist class	0.2	[8]
σ	Rate at which Terrorist become Combatant	0.3	[5]
δ	Rate of Jail break/Prisoners swap	01	[9]
k	Rate of Surrender Terrorist	0.2	Estimated
r	Rate at which detent member are rehabilitated	0.4	[3]
$1-\sigma$	Detention due to Counter Terrorist Strategies	0.7	[1]
ω	Radicalization/ rehabilitation rate	0.2	[7]
d	Induced death rate due military operation	0.3	[6]
μ	Natural death of Boko Haram member	0.1	Estimated

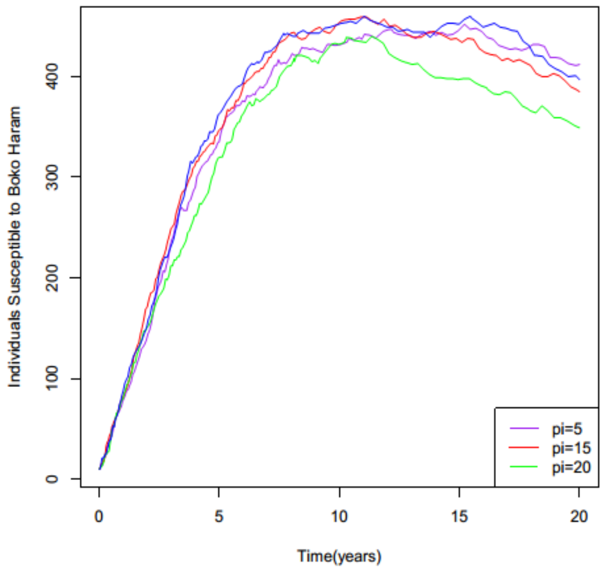

Fig 1: Sample path showing susceptible individuals over time at different values of recruitment rate (Pi).

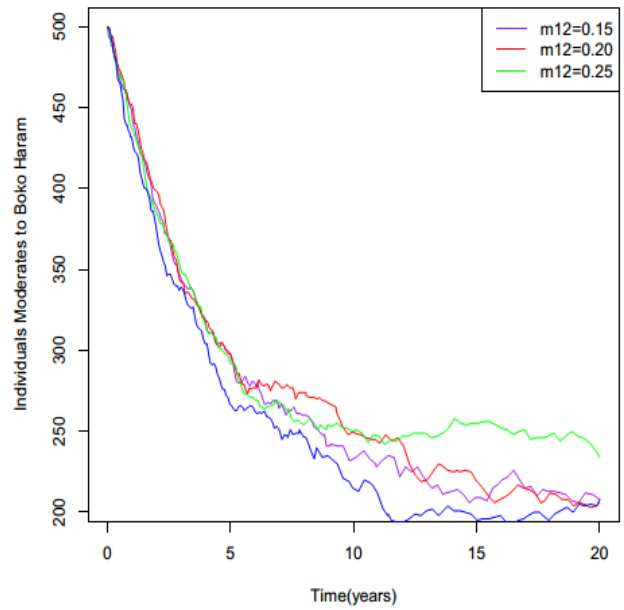

Fig 2: Sample path showing Moderate individuals over time at different values of m_{12}.

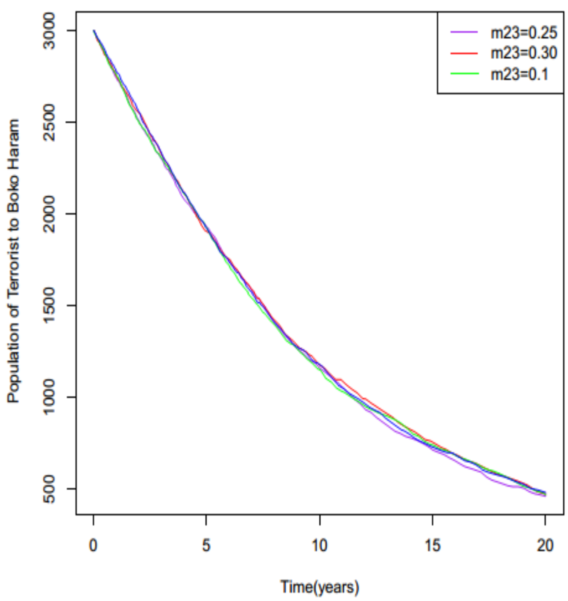

Fig 3: Sample path showing Terrorized individual over time at different values of m_{23}.

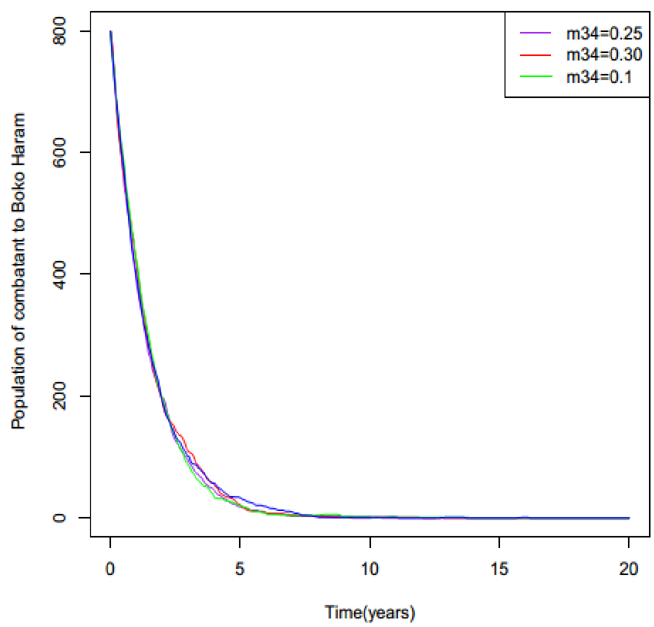

Fig 4: Sample path showing Combatant Terrorist Individual Over time at different values of m_{34}.

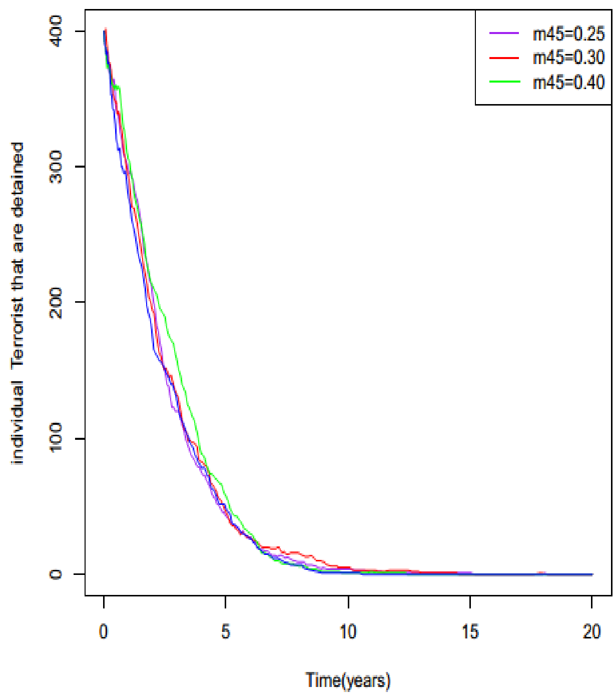

Fig 12: Sample path showing Detained Terrorist individual over time at different values of m_{45}.

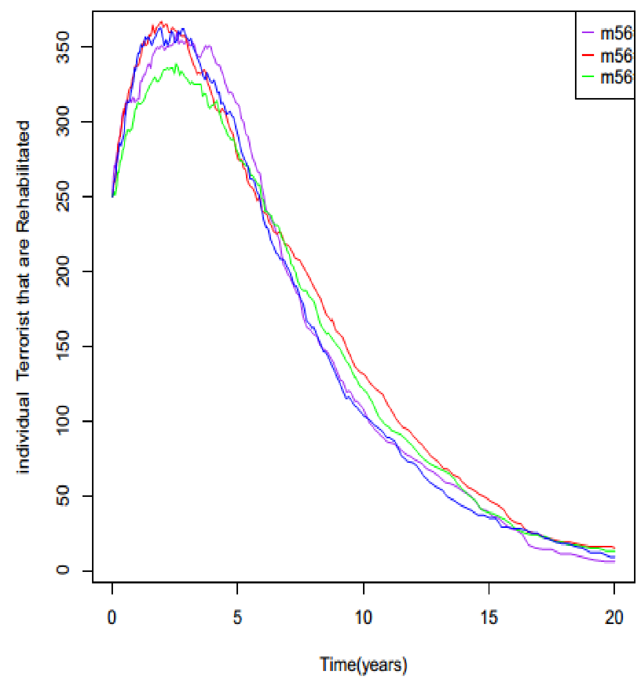

Fig 13: Sample path showing Rehabilitated Terrorist Individual over time at different values of m_{56}.

According to the results shown in **Figure 1,** the number of susceptible individuals, who are at risk of adopting extremist ideologies from the general population, tends to grow exponentially over time. This increase is attributed to their interactions with local communities in search of sympathizers or individuals who can assist them in facilitating attacks and spreading their ideology. In **Figure 2**, it is noted that moderate individuals, who are not actively involved but have some interest in Boko Haram activities due to their connections with terrorists, experience a gradual decline over time. This decline is linked to the interventions of security forces and civilian Joint Task Forces, which monitor and control their potential progression to active membership. **Figure 3** indicates that the number of Boko Haram terrorists is also decreasing over time, primarily due to counter-terrorism operations and the transfer of individuals to detention facilities if they surrender or are captured by military counter-terrorism efforts. The transition to extremism among these individuals is influenced by their interactions with terrorist leaders or combatants. Additionally, **Figure 4** shows that the number of Boko Haram combatants continues to decline over time, again as a result of counter-terrorism operations and the movement of individuals to detention facilities following surrender or capture.**Figure 5** shows the number of detained Boko Haram terrorist.

The detention members tend also decline this was due to high military counter-terrorist activities that accounted for individuals to be imprisoned for life.**Figure 6,** the number of Rehabilitated Boko Haram tends to raises at some point and then later decline over time. This was as a result of the population decreases to natural death and certified repentant terrorist also leave this population to be incorporated back to the society and the certified repentance move from rehabilitation to susceptible.

4 CONCLUSION

We developed and analyzed a stochastic model to understand the dynamics and management of the Boko Haram insurgency in conjunction with counter-terrorism operations. The differential equations that form the basis of the model were established, and simulations were carried out using the available parameters. The results indicated that counter-terrorism strategies have a substantial effect on combating Boko Haram insurgencies. Consequently, we recommend that the Nigerian Army, with significant support from the Air Force, intensify their efforts to overcome the Boko Haram insurgency, including the acquisition of modern weaponry, the establishment of vigilante groups, and the implementation of rehabilitation programs to maintain the defeat of Boko Haram.

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